MODELING OF TEMPERATURE PROCESS SYSTEM AND ITS PERFORMANCE ANALYSIS USING VARIOUS CONTROL STRATEGIES

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ABSTRACT

A Continuous Stirred Tank Reactor (CSTR) is a highly non-linear process particularly when chemical reaction takes place. The control of temperature for this process is a real challenge due to non-linear temperature changes during reaction. This paper compares the performances of the Proportional Integral Derivative Controller (PID) controller, Neural Network Predictive Controller (NNPC) and Non Linear Auto Regressive Moving Average (NARMA) controller. A novel NARMA based PID controller is proposed. The mathematical model of CSTR is obtained and the state space model is derived. The various controllers have been designed and performances were compared for the CSTR process. The proposed NARMA based PID controller shows better control of temperature than the other controllers like PID, NARMA and NNPC.

Index Terms - CSTR, PID, NNPC, NARMA, NARMA-PID

I. INTRODUCTION

Chemical reactors are often the most difficult units to control in a chemical plant, particularly if the reactions are rapid and exothermic. A continuous stirred reactor with a constant feed rate, feed concentration, and holdup time, with irreversible exothermic reaction is considered. The heat generated by chemical reaction, the heat removed by the jacket and the product stream temperature are plotted against reactor temperature to show three possible operating states [1]. The amount of heat released by exothermic reaction is sigmoidal function of temperature in the reactor. The heat removed by the coolant is linear function of temperature. The intersection of the curves yields three states [2]. A CSTR at steady state will have the heat generated by reaction is equal to heat removed by the coolant. A controller that ensures the stability of the operation at the middle steady state is desirable. The control of non isothermal CSTR using PID, IMC controller [3] gives basic control strategies. The implementation of neural control [4] of CSTR is also detailed. Figure 1 shows a CSTR in which an irreversible exothermic reaction A→B takes place. The heat of reaction is removed by a coolant medium that flows through a jacket around the reactor.

![Figure 1. CSTR with cooling jacket](image)

II. MODEL DESCRIPTION

The dynamic model of the reactor is obtained by writing material and energy balance equations. The component balance equation assuming constant inlet and outlet flow rate (F), and density (ρ) is given below

\[ \frac{dC_A}{dt} = F \cdot (C_{A_0} - C_A) - C_A - V \cdot \beta \rho = \alpha \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]  

(1)

The energy balance equation assuming constant volume (V), heat capacity (C_p) and density (ρ) is given below

\[ \text{Energy Accumulation} = \text{(Energy in} - \text{Out by flow)} + \text{Heat of reaction Contribution} - \text{Heat Transferred to Jacket} = 0 \]

\[ \frac{dH}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

(2)

The change in concentration of the reactant and temperature of the reactor is mathematically written as [5] given below.

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

(3)

The non-linear term present in the above modeling equations, \( \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \) is linearized close to the operating point \( T_0 \) and \( C_{A_0} \) using Taylor’s series expansion as given below

\[ \exp \left( \frac{-E_a}{RT} \right) \approx 1 - \frac{E_a}{RT} \]

(4)

The linearized model is obtained, after substitution of above equation to the dynamic model as given below

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

(5)

The linearized model is obtained, after substitution of above equation to the dynamic model as given below

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

(6)

\[ \frac{dC_A}{dt} = \frac{F}{V} \cdot (C_{A_0} - C_A) - Q_i \cdot \exp \left( \frac{-E_a}{RT} \right) \cdot C_A \]

(7)
The steady state condition is obtained when the two state derivatives are set equal to zero.

\[ \frac{dx}{dt} = 0, \frac{du}{dt} = 0 \]  

(8)

The generic state space model is of the form.

\[ x = Ax + Bu, y = Cx + Du \]  

(9)

The state and input variables represented in deviation variable form are as follows.

\[ x = \begin{bmatrix} [A]_{11} & [A]_{12} & \cdots & [A]_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ [A]_{m1} & [A]_{m2} & \cdots & [A]_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, u = \begin{bmatrix} T_f - T_i \\ \vdots \\ T_f - T_i \end{bmatrix} \]  

(10)

The jacket temperature alone is manipulated variable. The remaining variables temperature of feed component, concentration, and feed flow rate are disturbances. The CSTR is modeled with parameters [6] given in Table 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_a)</td>
<td>Activation Energy</td>
<td>32400 Btu/lb.Mol</td>
</tr>
<tr>
<td>(k_0)</td>
<td>Arrhenius factor or Pre Exponential Factor</td>
<td>15e12 per Hr</td>
</tr>
<tr>
<td>(\Delta H)</td>
<td>Heat of Reaction</td>
<td>-45000 Btu/lb.Mol</td>
</tr>
<tr>
<td>(U)</td>
<td>Heat Transfer Coefficient</td>
<td>75 Btu/ft² °F</td>
</tr>
<tr>
<td>(\rho \cdot c_p)</td>
<td>Density (\times) Specific Heat Capacity</td>
<td>53.25 Btu/ft²</td>
</tr>
<tr>
<td>(R)</td>
<td>Gas Constant</td>
<td>987 Btu/lb.Mol °F</td>
</tr>
<tr>
<td>(V)</td>
<td>Volume of Reactor</td>
<td>750 Ft³</td>
</tr>
<tr>
<td>(F)</td>
<td>Flow Rate of Coolant</td>
<td>3000 Ft³/Hr</td>
</tr>
<tr>
<td>(C_{A,f})</td>
<td>Concentration of Component A in feed</td>
<td>0.132 lb.Mol/Ft³</td>
</tr>
<tr>
<td>(T_i)</td>
<td>Temperature of the feed</td>
<td>60 °F</td>
</tr>
<tr>
<td>(A)</td>
<td>Heat Exchange Surface Area</td>
<td>1221 Ft²</td>
</tr>
</tbody>
</table>

The state space model obtained [7] for the reactor with operating point concentration and temperature of 0.08 lb.Mol/Ft³ and 80 °F is given below.

\[ A = \begin{bmatrix} 1.0119 & -1.0119 & 0 \\ 1.0119 & -1.0119 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.0014 \\ 0.0014 \\ 0 \end{bmatrix} \]  

(11)

Substituting the parameters from the table 1 to the above matrix with \(T_f = 460°\) Rankine and \(C_{A,f} = 0.08\) lb.mol/ft³, the obtained state matrix is

\[ A = \begin{bmatrix} 4.0066 & 5.10704 & -6.2615 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \]  

(12)

The input matrix is given as below.

\[ B = \begin{bmatrix} 0.0014 \\ 0.0014 \end{bmatrix} \]  

(13)

Substituting the parameters from the table 1 to the above matrix with \(T_f = 460°\) Rankine and \(C_{A,f} = 0.08\) lb.mol/ft³, the obtained input matrix is

\[ B = \begin{bmatrix} 0.0014 \\ 0.0014 \end{bmatrix} \]  

(14)

The output matrix is an identity matrix that takes only the temperature as the variable desired. It neglects the concentration.

\[ C = \begin{bmatrix} 1 & 0 \end{bmatrix} \]  

(15)

The disturbance matrix represents both concentration and temperature of feed variation. This is assumed to be negligible so the disturbance matrix is assumed to be zero.

\[ D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]  

(16)

The initial reactor concentration and temperature are 0.1 lb/ft³ and 40 °F.

III. CONTROL STRATEGIES

A. Proportional Integral Derivative (PID) Controller

The PID controller has been used for the Temperature and Concentration control for CSTR over past two decades. The time domain representation of PID control is

\[ u(t) = K_c (e(t) + \frac{1}{T_i} \int_0^t e(t) dt + \frac{T_d}{T_i} \frac{de(t)}{dt}) \]  

(17)

Where \(T_i\) is integral time and \(T_d\) is derivative time. The initial \(K_p, K_i, K_d\) values of the parameter are chosen as 10, 100, 0.001 respectively for the PID Controller. The PID parameters are found using Zeigler –Nichols method. The block diagram used for the simulation of the closed loop response of state space model of the plant with and without PID Controller is shown in Figure 2.

Fig. 2. CSTR Simulink Block with and without PID Controller

Fig. 3. Response with and without PID Controller
The closed loop response of plant model with and without PID controller is given in Figure 3. A set point of 80 °F is given as step input. The difference between set point and the measured temperature i.e., the error is given as an input to the PID controller. The delay time and rise time is 0.0286 Sec and 0.0737 Sec respectively. For the system with PID Controller the settling time is around 0.0916 Sec with little overshoot of 4.5125% in the response. The steady state error is negligible for the plant with PID controller. For a plant without controller the temperature settles at a lower stable value.

B. Neural Network Predictive Controller (NNPC)

A plethora of Model Predictive Controller for control of temperature, concentration, Ph without neural strategy for CSTR is given in [8,9,10]. The Neural network Predictive Controller uses a neural network model to predict future plant responses to relevant control signals. An optimization algorithm then computes the control signals that optimize future plant performance. The neural network plant model is trained offline, in batch form. The controller, however, requires a significant amount of online computation, because an optimization algorithm is performed at each sample time to compute the optimal control input. The first stage of predictive control is to train a neural network to represent the forward dynamics of the plant. The prediction error between the plant output and the neural network output is used as the neural network training signal. The process is represented by the following Figure 4.

\[
\begin{align*}
N1, N2, \text{and } Nu & \text{ define the horizons over which the tracking error and the control increments are evaluated. The } u' \text{ variable is the tentative control signal, } y_d \text{ is the desired response, and } y_m \text{ is the network model response. The } p \text{ value determines the contribution that the sum of the squares of the control increments has on the performance index. The following block diagram illustrates the model predictive control process. The controller consists of the neural network plant model and the optimization block. The optimization block determines the values of } u' \text{ that minimize } J, \text{ and then the optimal } u \text{ is input to the plant. The control literature } [11, 12, 13, 14] \text{ have proposed neural network based model predictive control for non linear CSTR process. The structure of NNPC is shown in Figure 5.}
\end{align*}
\]

The two steps involved when using neural networks for control system are System identification and Control design. In the system identification stage, the neural network model of the plant to be controlled is developed. In the control design stage, the neural network plant model is used to design the controller. The advantage of using Artificial Neural Networks to simulate the process is that after they are trained, they represent a quick and reliable way of predicting their performance. They can also be continuously updated. The simulink block diagram of CSTR with NNPC is shown in Figure 6.

The training, testing and validation data for neural network model in NNPC is shown in Figures 7a,b,c.
The Mean Square Error parameter is used as the performance indices of NNPC is shown in Figure 8, for the above data.

![Fig. 8. The Mean Square Error performance of NNPC](image)

The closed loop response of plant model with NNPC is given in Figure 9. A set point of 80°F is given as reference input at unit time instant. For the plant with NNPC, the settling time is around 1.1594 Sec with over shoot of 12.5% is in the response. The steady state error is negligible for the plant with NNPC. The delay time, rise time is 0.4188 Sec and 0.3347 Sec respectively.

![Fig. 9. Response of plant with NNPC](image)

### C. Non Linear Auto Regressive Moving Average (NARMA) Controller

The NARMA model is an exact representation of the input output behavior of non-linear dynamic systems. NARMA Controller requires the least computation. The controller is simply a rearrangement of the neural network plant model, which is trained offline, in batch form. The only online computation is a forward pass through the neural network controller. The drawback of this method is that the plant must either be in companion form, or be capable of approximation by a companion form model. As with model predictive control, the first step in using feedback linearization (or NARMA) control is to identify the system to be controlled. Train a neural network to represent the forward dynamics of the system. The first step is to choose a model structure to use. One standard model that is used to represent general discrete-time nonlinear systems is the nonlinear autoregressive-moving average (NARMA) model [15]:

\[
y(k) = \sum_{i=1}^{p} a_i y(k-i) + \sum_{i=1}^{q} b_i u(k-i) + \epsilon(k)
\]

Where,

\[ u(k) \text{ is the system input and } y(k) \text{ is the system output. For the identification phase, a neural network is trained to approximate the nonlinear function } N. \]

This is the identification procedure used for the NN Predictive Controller. If the system output has to follow some reference trajectory

\[
y(k + d) = y_r(k + d), \tag{20}
\]

The next step is to develop a nonlinear controller of the form:

\[
y(k + d) = y_r(k + d).
\]

The problem with using this controller is that if we want to train a neural network to create the function \( G \) to minimize mean square error, use dynamic back propagation. This can be quite slow. One solution is to use approximate models to represent the system. The controller used in this section is based on the NARMA approximate model:

\[
y(k + d) = y_r(k + d).
\]

This model is in companion form, where the next controller input \( u(k) \) is not contained inside the nonlinearity. The advantage of this form is that we can solve for the control input that causes the system output to follow the reference \( y(k + d) = y_r(k + d) \). The resulting controller would have the form

\[
y(k + d) = y_r(k + d).
\]

Using this equation directly can cause realization problems, so determine the control input \( u(k) \) based on the output at the same time, \( y(k) \). So, instead, use the model

\[
y(k + d) = y_r(k + d).
\]

Where, \( d \geq 2 \). Using the NARMA model, we can obtain the controller

\[
y(k + d) = y_r(k + d).
\]

This is realizable for \( d \geq 2 \). The block diagram of the NARMA controller [16] is shown in Figure 10.

![Fig. 10. NARMA Structure with Plant Model](image)

The simulink block to obtain the response of plant with NARMA Control is shown in Figure 11.
The training, testing and validation data for neural network model in NARMA controller is shown in Figure 12a,b,c.

The Mean Square Error is used as the performance indices of NARMA controller and it is shown in Figure 13.

The artificial neural network is used to develop better and more efficient nonlinear CSTR modeling and control [17, 18]. The response of the CSTR model with NARMA Controller is shown in Figure 14. It has delay time, rise time of 0.0392 Sec and 0.0726 Sec respectively. It has no overshoot and steady state error. The settling time is around 0.0857 Sec.

The response of Plant with NARMA-PID is shown in Figure 16. It has delay time, rise time and settling time of 0.0038 Sec, 0.0120 Sec and 0.0152 Sec respectively. It has little over shoot of 1.466% and no steady state error.

IV. RESULTS & DISCUSSION

The comparative responses of the plant for a reference input of 80°F given at time t =1Sec is plotted for the PID, NNPC, NARMA and the Proposed NARMA-PID Controllers are given in Figure 17. The delay time is the time taken to reach 50% of the final steady state output i.e. 40°F. The rise time is the time taken for the response to rise from 10% to 90% of the steady state value, i.e. from 84°F to 72°F. The settling time is the time taken for the response to be within ±5% of final steady state value i.e. 84°F or 76°F. The response of the plant without controller settles around 22°F.
The inference obtained from the response is shown in Table 2.

**TABLE II. COMPARATIVE ANALYSIS OF CSTR USING VARIOUS CONTROLLERS**

<table>
<thead>
<tr>
<th>Types of Controller</th>
<th>Delay Time $t_d$ in Sec.</th>
<th>Rise Time $t_r$ in Sec.</th>
<th>Settling Time $t_s$ in Sec.</th>
<th>Peak Overshoot $M_p$ in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>0.0286</td>
<td>0.0737</td>
<td>0.0916</td>
<td>4.5125</td>
</tr>
<tr>
<td>NNPC</td>
<td>0.4188</td>
<td>0.3347</td>
<td>1.1594</td>
<td>12.5</td>
</tr>
<tr>
<td>NARMA</td>
<td>0.0392</td>
<td>0.0726</td>
<td>0.0857</td>
<td>Nil</td>
</tr>
<tr>
<td>Proposed NARMA-PID</td>
<td>0.0038</td>
<td>0.0120</td>
<td>0.0152</td>
<td>1.466</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper the performance of Proportional Integral Derivative (PID) Controller, Neural Network Predictive Controller (NNPC) and Non Linear Auto Regressive Moving Average (NARMA) Controller performances are compared. It is found that both the settling time and rise time of NNPC is higher compared to conventional PID and NARMA. The peak overshoot of NNPC is higher than Conventional PID. The NARMA has no overshoot and least overshoot. The delay time, rise time, and settling time. for the proposed NARMA-PID is the least of all controllers. From the extensive simulation study using MATLAB / Simulink software [19], it is found that for Non linear systems such as CSTR process the NARMA-PID Controller’s performance is better than PID, NNPC and NARMA controller.

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REFERENCES